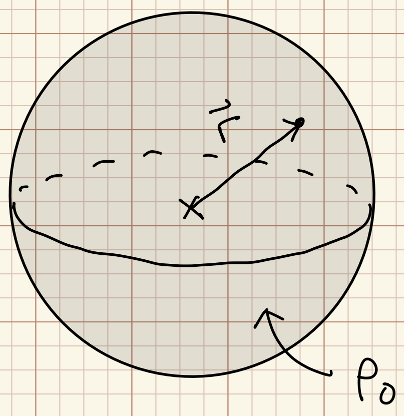
▨ An Off-Center Spherical Cavity

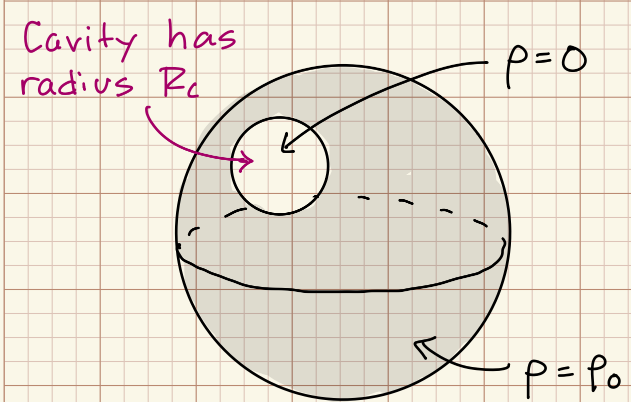
- Inside a uniform ($\rho = \rho_0 = \text{constant}$) solid sphere, the electric field is



$$\vec{E}(r<R) = \frac{\rho_0 r}{3\epsilon_0}\hat{r}$$ ← Did this in class with Gauss's Law!

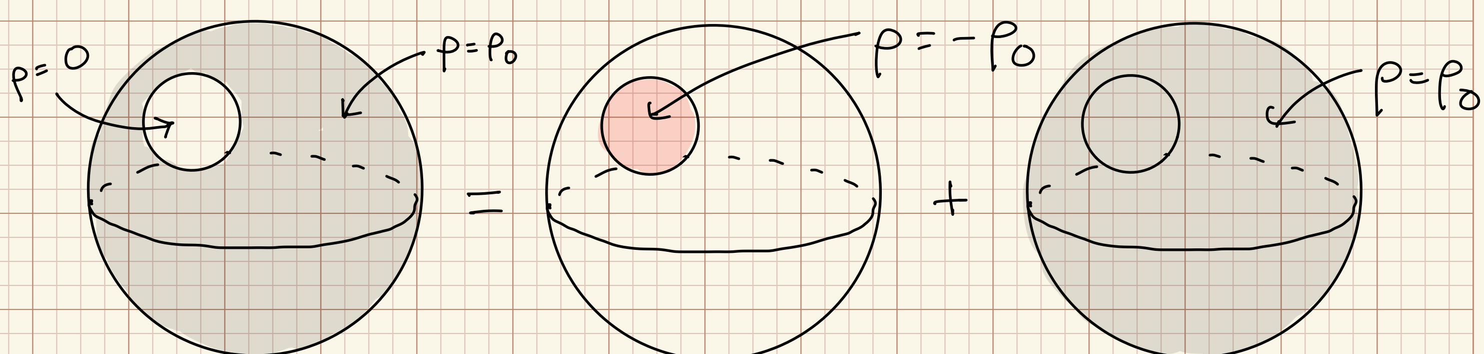
$$\vec{E}(r<R) = \frac{\rho_0}{3\epsilon_0}\vec{r}$$ ← Because $r\hat{r} = \vec{r}$

- What if we hollow out a spherical cavity that is off center? What is $\vec{E}$ inside the cavity?

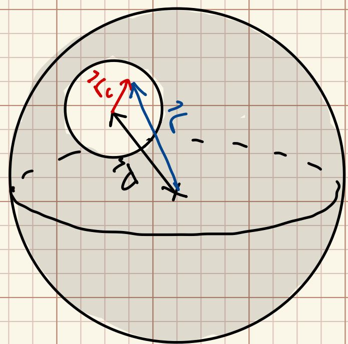


← This is not spherically symmetric! So we cannot conclude $\vec{E} = 0$ inside cavity, even though any Gaussian surface would enclose no charge.

- Even though this isn't spherically symmetric, it is the sum of two spherically symm. charge distributions!



- Describe the offset by a vector $\vec{d}$ pointing from the center of the sphere to the center of the cavity:



A point inside the cavity located @ $\vec{r}_c$ from center of cavity has pos. $\vec{r} = \vec{d} + \vec{r}_c$ from center of sphere.

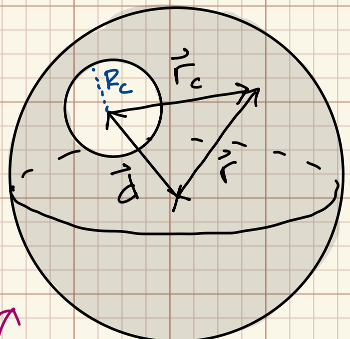
- By <u>superposition</u>, $\vec{E}$ inside cavity is

$$\vec{E} = \frac{\rho_0}{3\epsilon_0}\vec{r} + \frac{(-\rho_0)}{3\epsilon_0}\vec{r}_c = \frac{\rho_0}{3\epsilon_0}(\vec{r}-\vec{r}_c) = \frac{\rho_0\vec{d}}{3\epsilon_0}$$

$$\hookrightarrow \boxed{\vec{E} = \frac{\rho_0\vec{d}}{3\epsilon_0}}$$

$\vec{E}$ is <u>constant</u> inside the cavity, so $\vec{\nabla}\cdot\vec{E} = 0$ ✓

- What about inside the sphere but outside the cavity? Pos. relative to center of $-\rho_0$ sphere is "$\vec{r}_c$"



$\vec{r} = \vec{d} + \vec{r}_c$

$\vec{r}_c = \vec{r} - \vec{d}$

Cavity has radius R_c

$$\vec{E} = \frac{\rho_0\vec{r}}{3\epsilon_0} + \frac{(-\rho_0)}{3\epsilon_0}\frac{R_c^3}{r_c^2}\hat{r}_c$$

$$\hat{r}_c = \frac{\vec{r}_c}{r_c}$$

$$= \frac{\rho_0\vec{r}}{3\epsilon_0} - \frac{\rho_0}{3\epsilon_0}\frac{R_c^3}{r_c^3}\vec{r}_c$$

$$\boxed{\vec{E} = \frac{\rho_0}{3\epsilon_0}\left(\vec{r} - \frac{R_c^3}{|\vec{r}-\vec{d}|^3}(\vec{r}-\vec{d})\right)}$$

Not as simple, but still easy to work out!

- Here are some questions to consider.

(1) If $\vec{d} = 0$ this is a spherical shell w/ constant ρ_0 between inner radius R_c & outer radius R. Does it agree w/ what we're done elsewhere?

(2) What if the cavity had ρ_c constant but not 0? Could you determine $\vec{E}$ inside the cavity? Is it as simple as the hollow cavity?

(3) Does the cavity have to be fully inside the sphere? Why or why not?

(4) For the example we just looked at, what is $\vec{E}$ all the way outside the sphere? Does it do what you expect when $r \gg R$?